Pallister Park Primary School

How we teach calculations:

Moving Towards Mastery Policy for Mathematics

September 2020

Our maths curriculum:

Our intent is to produce an **ambitious, connected curriculum** accessible to **all pupils** in school right through from EYFS to the end of Y6. Our curriculum will not only cover all the content of the National Curriculum, but also provide advice and exemplification for teachers and parents.

As a school we want pupils to become fluent in the fundamentals of mathematics, to be able to reason and to solve problems. Our curriculum embraces these National Curriculum aims, and provides guidance to help pupils to:

- Use the Concrete Pictorial Abstract approach (detailed on p.3) to help pupils understand mathematics and to make connections between different representations.

- Develop mathematical language and questioning so pupils can discuss the mathematics they are doing, and support them to take ideas further.

- Become fluent mathematicians, and foster a love of learning for mathematics.

Why we use the White Rose Maths curriculum and follow an **order of progression**?

To learn mathematics effectively, **some things have to be learned before others**, e.g. place value needs to be understood before working with addition and subtraction, addition needs to be learnt before looking at multiplication (as a model of repeated addition).

You will see this emphasis on number skills first, carefully ordered, throughout our primary curriculum. For some other topics, the order isn't as crucial, e.g. Shapes and Statistics need to come after number, but don't depend on each other. We mix these so pupils have as wide a variety of mathematical experiences as possible in each term and year.

The following calculation policy has been revised in September 2020 to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in Reception follows the '**Development Matters**' EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age stage expectations

Maths teaching across the school now incorporates a 'Mastery' approach to learning from Year 1 to Year 5 in 2020/21. This reflects national requirements to allow all children to access skills, methods and concepts within their year group. Children are assessed against year group expectations leading to their mastery of skills over the academic year. At the end of blocks of work children will be deemed to be working towards, meeting the national expectations or working with greater depth. Within every year group, fluency is a high priority in order for a child to 'master' a concept. In order to provide depth of understanding, children will be given opportunities to problem solve, reason and explain.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons, although skills will still need careful teaching and practice. Staff are now using a 'mastery' approach to teaching which involves several stages to support a concept.

Choosing a calculation method:

During a lesson, children can explore and choose a concrete, pictorial or abstract way of working. Teachers will model all three methods of working to encourage children to practically use equipment, to visually see a concept, and then to apply the prior learning in order to utilise a more abstract method which involves using symbols, digits and working mentally.

Concrete Representation

The practical **stage**. Children are first introduced to an idea/skill/concept by physically acting it out with real objects, this could also involve the outdoor environment. This is a **'hands on'** stage using real objects linked to real-life and the wider curriculum and/or mathematical equipment, (e.g. five and ten frames, Dienes, Numicon etc.) and it is the prerequisite for a **CONCEPTUAL UNDERSTANDING**.

Pictorial Representation

The **image stage**. A child must of attained a sufficient understanding at the practical, **CONCRETE** experiences performed and can now relate them to **PICTORIAL** representations, such as a **DIAGRAM** or **PICTURES** of the problem. **PICTORIAL** representations, such as the bar model, can also be used to aid a child's understanding.

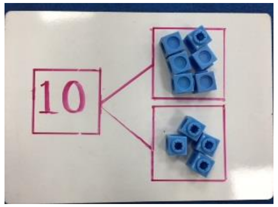
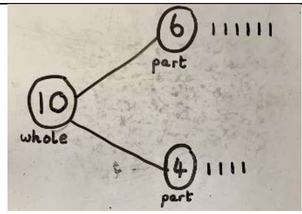
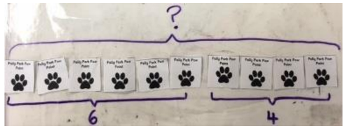
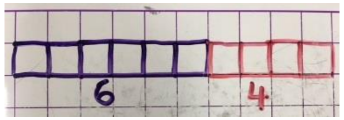
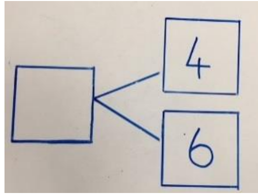
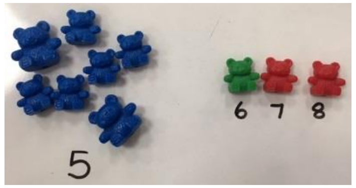
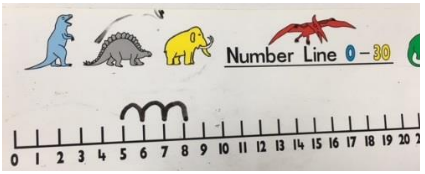
Abstract Representation

The **symbolic stage**. Children must be taught how to represent problems by using **ABSTRACT** mathematical notation, e.g. 12 x 2 = 24. The previous two methods of working out should facilitate the transition into enabling all children to complete abstract working out with understanding of what they are doing.

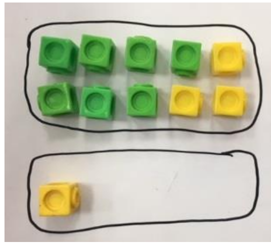
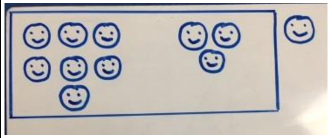
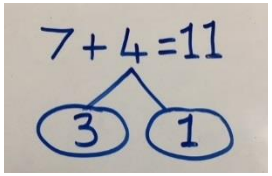
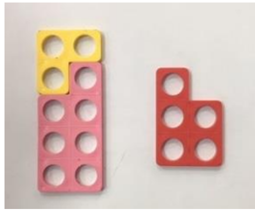
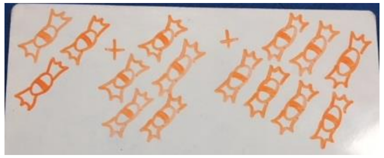
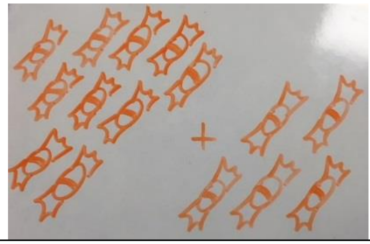
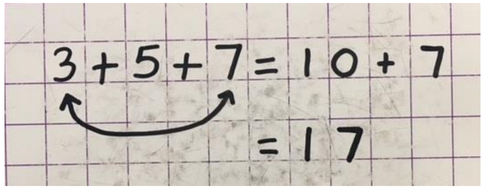
"If we do not use concrete manipulations, then we cannot understand mathematics. If we only use concrete manipulations, then we are not doing mathematics." Maths No Problem (2016)

To work out a calculation:
Concrete
Pictorial
Abstract

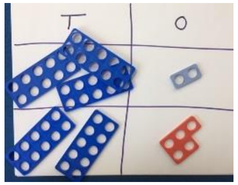
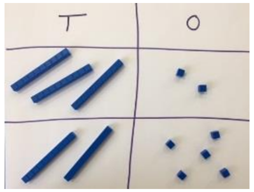
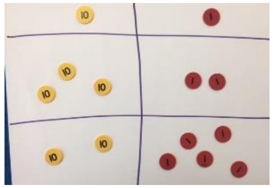
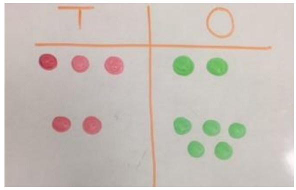
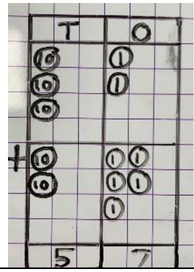
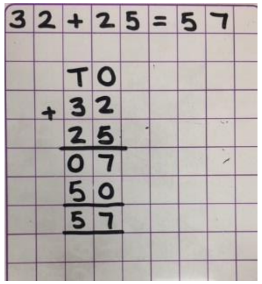
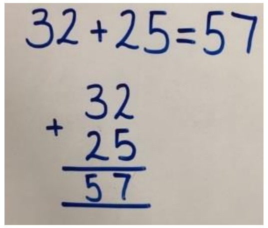
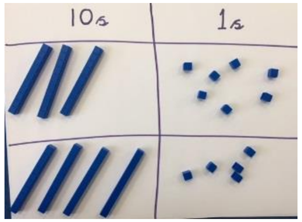
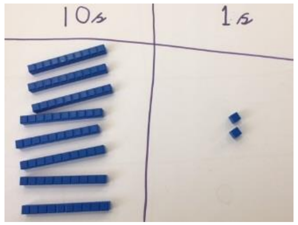
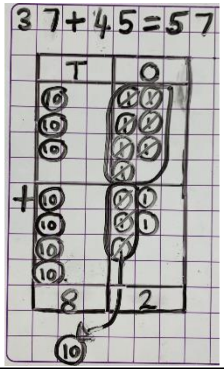
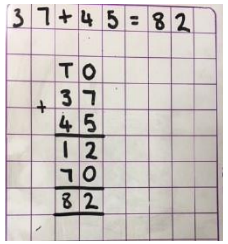
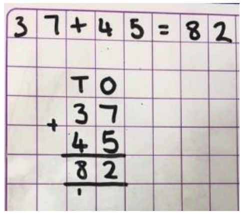
Addition

	Objective	Concrete	Pictorial	Abstract
Y1	Number bonds of 5, 6, 7, 8, 9 and 10	   Use cubes to add two single digit numbers together as a group or in a bar. Use of part whole models at this stage will scaffold future learning.	 Use pictures to add two single digit numbers together as a group or in a bar.  	6 + 4 = 10 4 + 6 = 10 10 = 6 + 4 10 = 4 + 6  Use the part-part-whole diagram as shown to move into abstract working.
Y1	Counting	 Start with the larger number on a bead or with items and count on the smaller number to find the answer. 	Use a number line to count on in ones. 	5 + 3 = 8 3 + 5 = 8 8 = 3 + 5 8 = 5 + 3

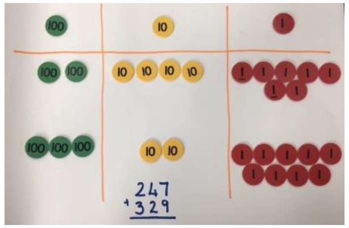
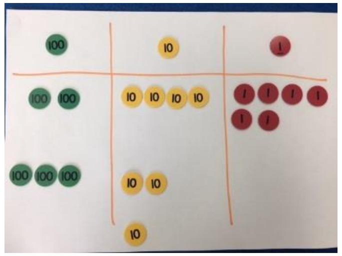
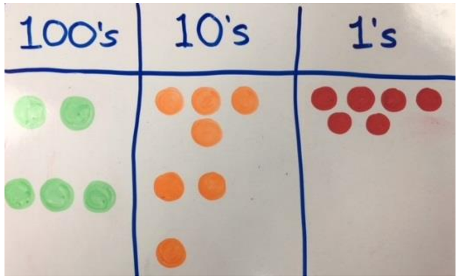
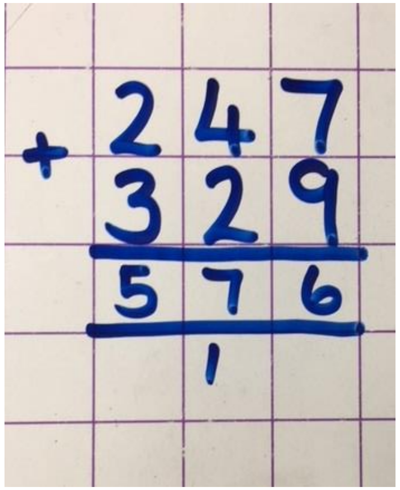
Addition

	Objective	Concrete	Pictorial	Abstract
Y1	Adding 2 single digit numbers (Regrouping to make 10)	7 + 4 = 11   Start with the bigger number and use the smaller number to make 10.	  7 + 3 = 10 10 + 1 = 11	**7 + 4 = 11**
Y2	Adding 3 single digit numbers	3 + 5 + 7 = 15 Put 3 and 7 together to make 10 if possible. Then add on the third digit.  	Add together three groups of objects. Draw a picture to recombine the groups to make 10.  	 Combine the two numbers that make 10 and then add on the remainder.

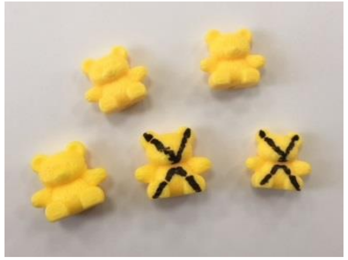
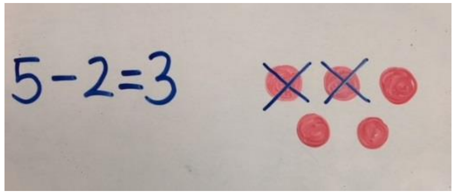
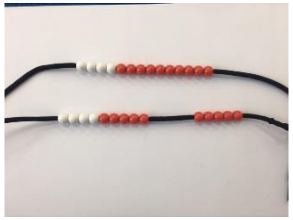
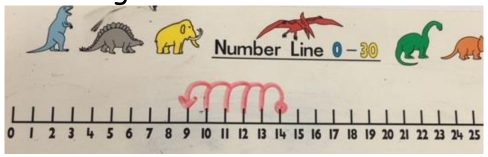
Addition

	Objective	Concrete	Pictorial	Abstract
Y2	Column method without regrouping	Add together the ones first, then add the tens. Use either Dienes/Numicon/10s/1s first before moving onto place value counters. 32 + 25 =   	After physically using apparatus and place value counters, children can draw the counters to help them solve additions.   	 Expanded method can be used to consolidate place value knowledge.  Efficiency achieved by moving onto the compact method.
Y2	Column method with regrouping	 Make both numbers on a place value grid.  Add up the ones, then exchange 10 ones for 1 ten.	Children can draw pictures of 10s/1s as above to help them to solve additions.  	 Expanded method.  Efficiency achieved by moving onto the compact method.

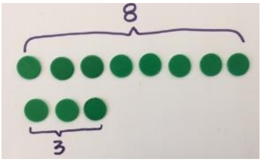
Addition

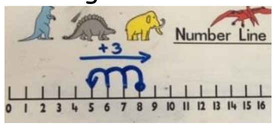
	Objective	Concrete	Pictorial	Abstract
Y3/4	Column method with regrouping. Y3 - 3 digit Y4 – 4 digit	Make both numbers on a place value grid.  Add up the ones and exchange the 10 ones for 1 ten.  As children move on to decimals, money and decimal place value counters can be used to aid understanding.	  Children can draw pictures of counters or Hs, Ts and Os transitioning from apparatus to support learning and understanding. *Addition of money to use £ and p separately in pictorial stage.	 *By Year Four children will progress on to adding four digit numbers. As children progress, decimals and money can be introduced here.
Y5/6	Column method with regrouping	Consolidation of understanding and using numbers with more than four digits and extending by adding numbers with up to three decimal places. *Practical apparatus available extends to numbers with up to three decimal places.		

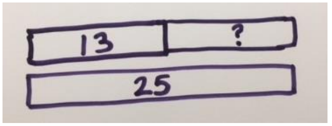
Subtraction

<table>
<tr><th></th><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td>Y1</td><td>Taking away ones</td><td>Using concrete apparatus such as cubes, counters etc. to show how objects can be physically taken away.
</td><td>Making use of drawn representations to show what has been taken away.

</td><td>5 - 2 = 3</td></tr>
<tr><td>Y1</td><td>Counting Back</td><td>
Make the largest number in the number sentence. Move beads along as children count back in ones.
14 - 5 = 9</td><td>Counting back on a number line.


Start at the largest number and count back showing the jumps on a number line.</td><td>Mental calculation of thinking of 14 and counting back 5. Use of fingers to help if needed.

14 - 5 = 9</td></tr>
<tr><td>Y1</td><td>Find the difference</td><td>Compare amounts and objects to find the difference.


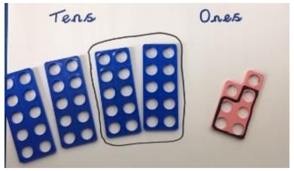
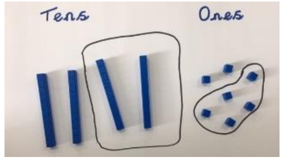
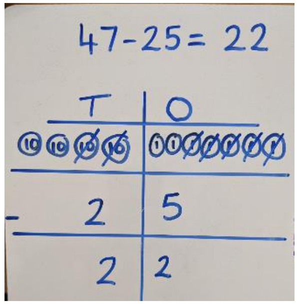
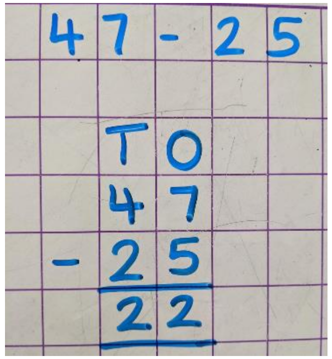
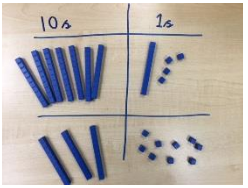
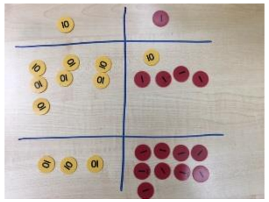
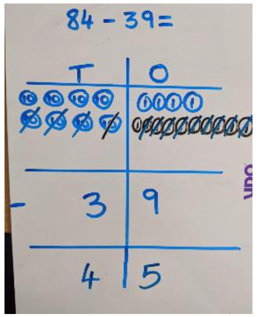
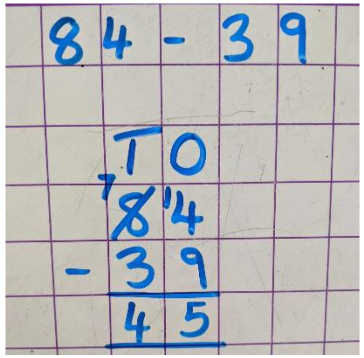
Use counters or cubes to make bars to find the difference. Basic bar models help to show the difference.</td><td>Counting on to find the difference.


Drawing bars to find the difference.

</td><td>Sarah has 8 marbles.
Tom has 3 marbles.

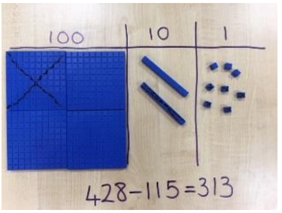
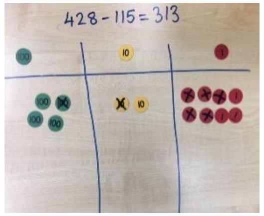
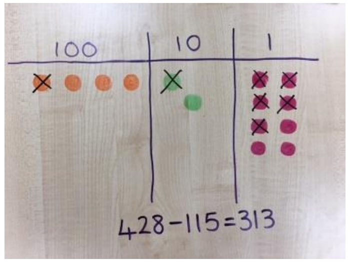
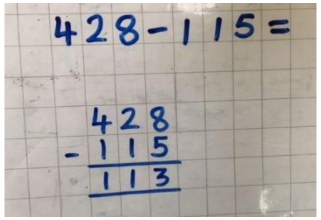
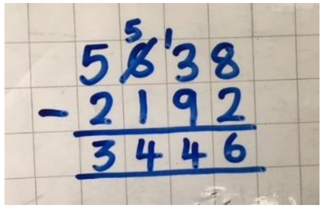
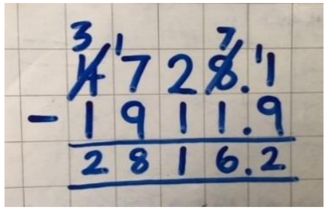
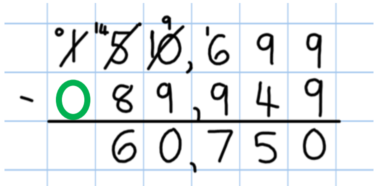
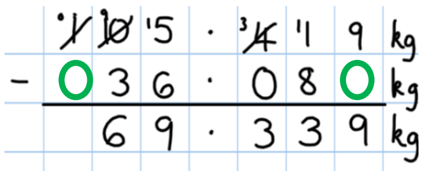
Find the difference between the number of marbles the children have.

8 - 3 = 5</td></tr>
</table>

Subtraction

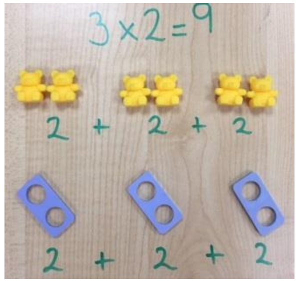
	Objective	Concrete	Pictorial	Abstract
Y2	Column method without regrouping	  Use apparatus – Dienes, Numicon or counters to make the bigger number then take the smaller number away. Counters show how to partition numbers to subtract. Again starting with the largest number.	Draw pictures of counters or 10s and 1s to represent the numbers – starting with the largest. These can be set out as they would be in columns. 	Practical and pictorial methods lead onto 'column subtraction'. 
Y2	Column method with regrouping	Starting with one exchange at Y2. By making the larger number with apparatus. Discuss and investigate "I need to exchange/move 1 of my tens into ten ones".  	Draw counters onto a place value grid and indicate what has been taken away and where exchanges have been made. 	Column subtraction is again used even when showing where values have been exchanged. 

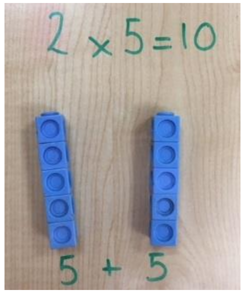
Subtraction

	Objective	Concrete	Pictorial	Abstract
Y3 Y4	Column method without regrouping *Y4 children will use similar methods with four digit numbers	Use Dienes/Base 10 to start with before moving onto place value counters. Revisit without regrouping before moving onto exchanging. 	Draw counters onto a place value grid and indicate what has been taken away and where exchanges have been made if needed. 	Practical and pictorial methods lead onto 'formal written method of compact column addition – fundamental to addition throughout the rest of Key Stage Two. 
Y4 Y5 Y6	Column method for subtraction	Compact column subtraction consolidates understanding with/without regrouping. Numbers contain more than four digits and extended by adding numbers with up to three decimal places. Zeros can been used to show the place value in each column. *Practical apparatus available extends to numbers with up to three decimal places. 		

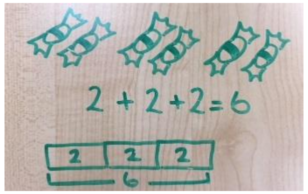
Multiplication

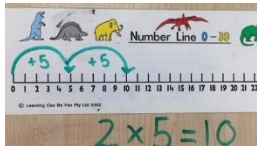
<table>
<tr><th></th><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td>Y1</td><td>Repeated addition</td><td>Make use of different objects to add equal groups together.



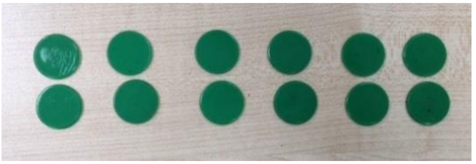


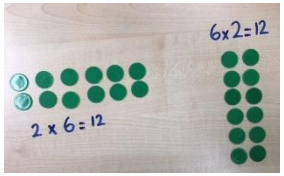
</td><td>Repeated addition can be used with pictures/number lines/bar models to illustrate problems.



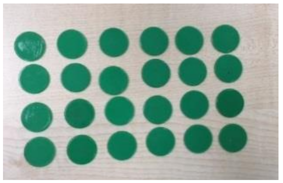


</td><td>Write addition sentences to describe objects and pictures.</td></tr>
<tr><td>Y1
Y2</td><td>Arrays – showing commutative multiplication</td><td>Create arrays using counters/cubes to show multiplication sentences.

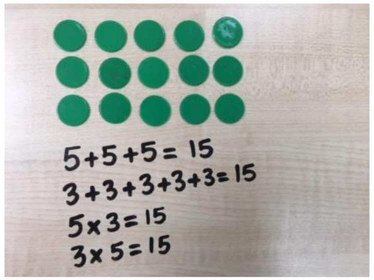

</td><td>Draw arrays in different rotations or bar models to illustrate commutative multiplication sentences.



6 x 2 = 12
2 x 6 = 12

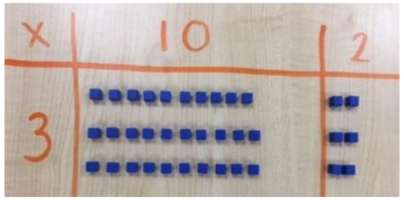
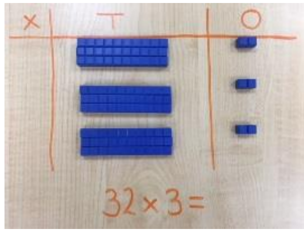
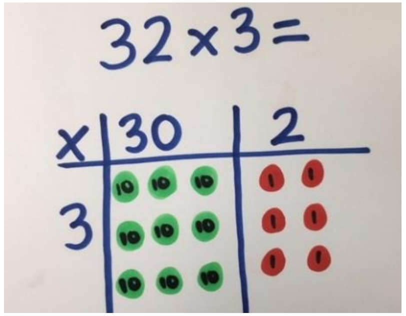
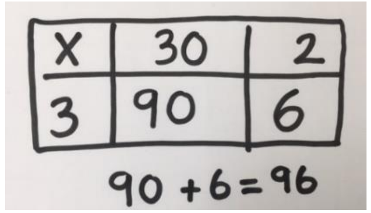
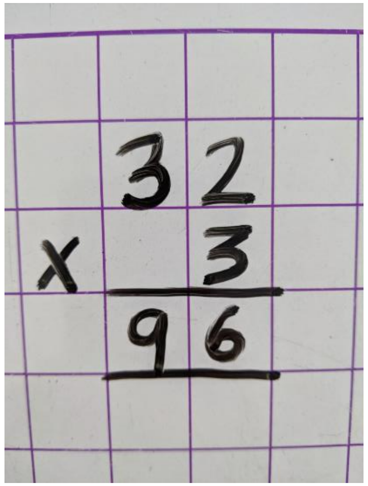


*Arrays can be linked with areas of rectangles.</td><td>Use of arrays can be used to compose more than one multiplication sentence and revise repeated addition.

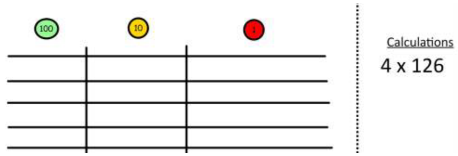
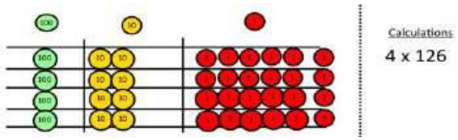
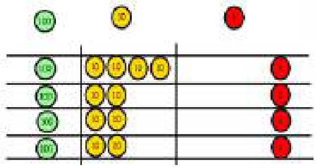
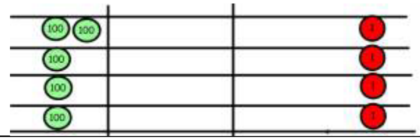
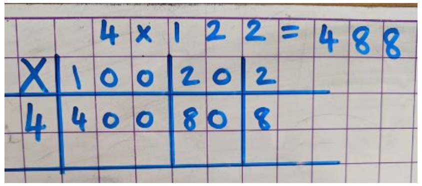
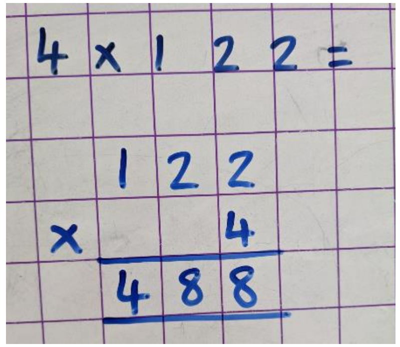


</td></tr>
</table>

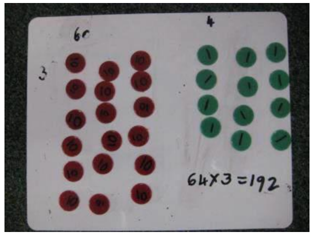
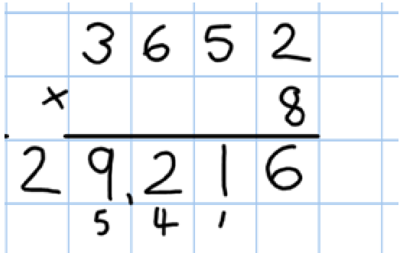
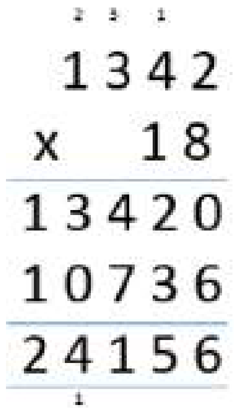
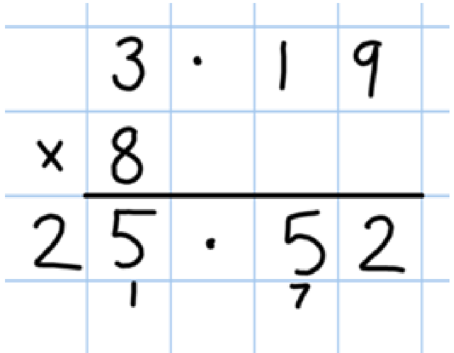
Multiplication

	Objective	Concrete	Pictorial	Abstract
Y3	Multiplication grid method 1 digit x 2 digit	Links with arrays to introduce the grid method by using counters. **(3 rows of 10 & 3 rows of 2).**  3 x 12 = 36 Move on to Dienes/Numicon/counters to move onto a compact method for larger problems. E.g. 3 x 32 = 96 **3 rows of 32.**  X 30 2 2 	Children can draw pictures to show the work they have done with place value counters and move away from practical resources in a way that they understand. Level of detail in pictures is not needed – a child's understanding of their representation is more important. 	Transition into abstract working is always the desired outcome. Start with multiplying by one digit numbers and showing the clear addition alongside the grid drawn.  

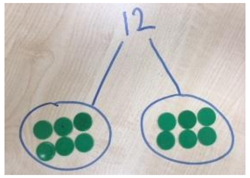
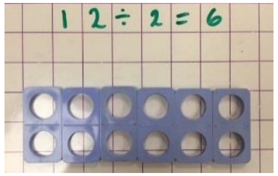
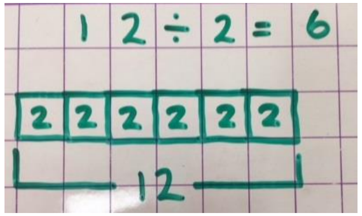
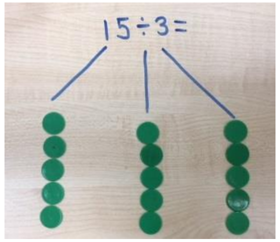
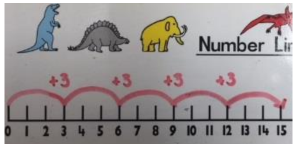
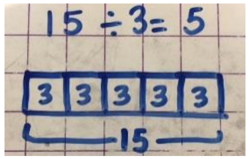
Multiplication

	Objective	Concrete	Pictorial	Abstract
Y3/Y4	Multiplication grid method progressing to column method (expanded) 1 digit x 2 digit (As shown in Y3) *1 digit x 3 digit	Progress on with place value counters to show how we are finding groups of a number. In this example, we are multiplying by 4 so we need 4 rows.  Fill each row with 126.  Add up each of the columns beginning with the 'ones' making exchanges where needed.  4 x 126 = 504 	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking. 	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. 

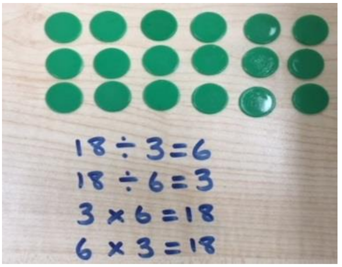
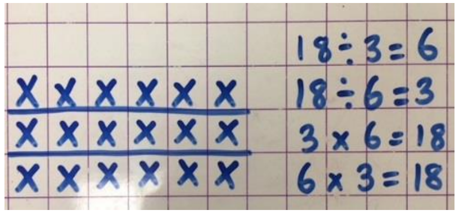
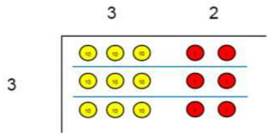
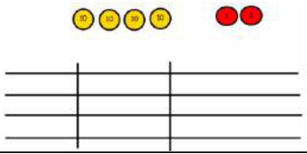
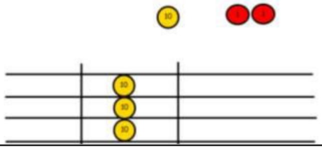
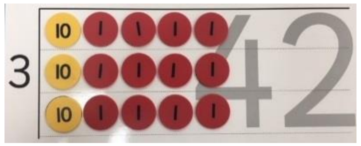
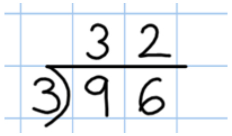
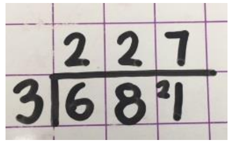
Multiplication

	Objective	Concrete	Abstract
Y5 Y6	Multiply 4 digit numbers by 1 or 2 digit numbers using formal methods	Where needed, children can continue to be supported by place value counters at the stage of multiplication. It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below. 	Revisit the expanded method as taught in Y3/Y4 **if necessary**, before progressing onto a compact method of working out for larger numbers. 3652 x 8 = 29,216 1342 x 18 = 24,156
Y6	Multiply one digit numbers with up to two decimal places by whole numbers		Use short multiplication (see Y5) to multiply numbers with more than 4-digits by a single digit; to multiply money and measures, and to multiply decimals with up to 2d.p. by a single digit.

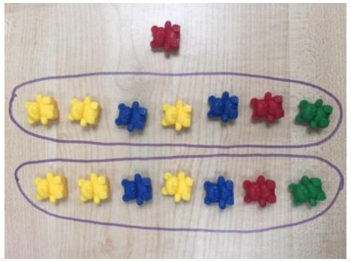
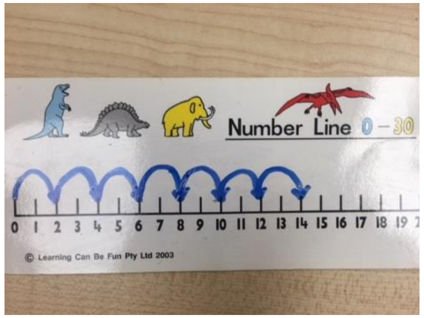
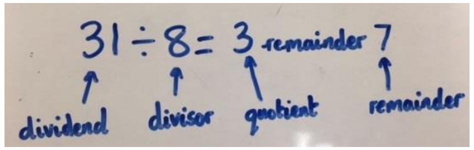
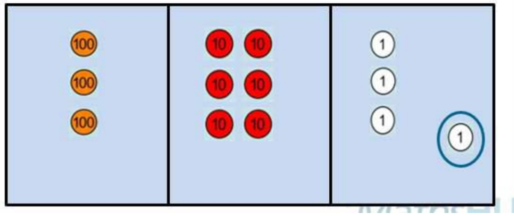
Division

	Objective	Concrete	Pictorial	Abstract
Y1 Y2	Sharing	I have 12 counters, can you share them equally between 2 people?   After making 12 with Numicon how can it be shared with the 2 plates?	Children can use pictorial representations or bar models to share out the quantities. 	Share 12 carrots between 2 rabbits… $12 \div 2 = 6$
Y2	Sharing and Grouping	Divide quantities into equal groups. Make use of counters, cubes, objects or place value counters to aid understanding.  	 Use a number line to represent jumps in groups. The number of jumps will equal the number of groups.  Use bars of 3 to make 15 and count how many bars were used.	$15 \div 3 = 5$

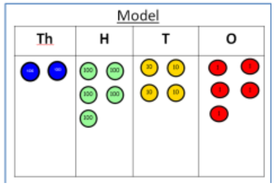
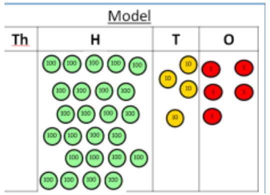
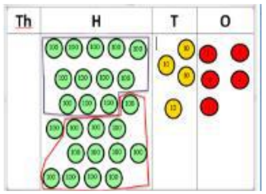
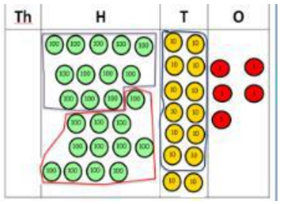
Division

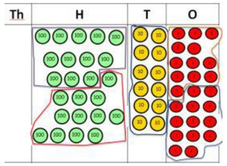
	Objective	Concrete	Pictorial	Abstract
Y2 Y3	Division with arrays	Make links from division to multiplication by creating an array and thinking of the number sentences that can be created. 	Draw an array and make use of lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating four linking number sentences. 18 ÷ 3 = 6 18 ÷ 6 = 3 3 x 6 = 18 6 x 3 = 18
Y3 Y4	Short Division	Make use of place value counters to divide using the short division method. (Counters can be replaced by pictures at the next stage, though may move swiftly into the written method dependent on understanding. Example without remainder **96 ÷ 3 = 32**  **42 ÷ 3 = 14** Start with the biggest place value. We are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over. We exchange this ten for 10 ones and then share the ones equally among the groups. We look at how many are in each group.   		Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., short division for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all.  

Division

	Objective	Concrete	Pictorial	Abstract
Y5 Y6	Division with remainders	Divide objects or items between groups and see how much is left over e.g. 15 ÷2 = 7 remainder 1 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 15 ÷2 = 7 remainder 1  	Complete written division statements and show the remainder using 'r'. 
Y5 Y6	Short division up to 4 digits by a single digit with remainders	364 ÷ 3 = $\begin{array}{r} 1\,2\,1 \text{ rem } 1 \\ 3 \overline{)\,3\,6\,4\,} \end{array}$  Use of place value counters to aid understanding.	Once competent in divisions with a remainder, children can begin to express as a fraction or a decimal according to the context. $\begin{array}{r} 1\;\;8\;\;6\;\tfrac{1}{5} \\ 5 \overline{)\,9\;{}^{4}3\;{}^{3}1\,} \end{array}$	$\begin{array}{r} 8\;\;6\;\;r\;2 \\ 5 \overline{)\,4\;{}^{3}3\;\;2\,} \end{array}$ $\begin{array}{r} 1\;\;4\;.\;6 \\ 35 \overline{)\,5\;\;1\;{}^{16}1\;.\;{}^{21}0\,} \end{array}$

Division

	Objective	Concrete	Abstract
Y6	Divide up to 4 digits by a two digit number Long division	 0 2 1 2 12 ⟌ 2544 24 14 12 24 24 0 2544 ÷ 12 How many groups of 12 thousands do we have? None  Exchange 2 thousand for 20 hundreds.  0 2 12 ⟌ 2544 24 1 How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.  0 2 1 12 ⟌ 2544 24 14 12 2 Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.	0 12 ⟌ 2544 First, exchange 2 thousand for 20 hundreds so we now have 25 hundreds. 0 2 12 ⟌ 2544 24 1 Secondly, how many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left. 0 2 1 12 ⟌ 2544 24 14 12 2 Now exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left. 0 2 1 2 12 ⟌ 2544 24 14 12 24 24 0 Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

		 Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2	